***Resolver***

$$1)\ \sum_{k=1}^{15}(3k^2-2k)$$

$$=\sum_{k=1}^{15}(3k^2-2k)=\sum_{k=1}^{15}3k^2-\sum_{k=1}^{15}2k=3\sum_{k=1}^{15}k^2-2\sum_{k=1}^{15}k$$

$$=3\cdot\frac{15(15+1)(2\cdot 15+1)}{6}-2\cdot\frac{15(15+1)}{2}=\frac{15(16)(31)}{2}-15(16)$$

$$=\frac{7440}{2}-240=3720-240=\boxed{\mathbf{3480}}$$

**2**) ***Encontrar el area bajo la curva de***

$f(x) = \sqrt{x+1}$ , *desde* $x = 0$ *hasta* $x = 8$ , *con* 6 *rectangulos*

$$\Delta x = \frac{8-0}{6} = \frac{8}{6} = \frac{4}{3}$$

$$A_1 = \frac{4}{3}\left(f\left(\frac{4}{3}\right) + f\left(\frac{8}{3}\right) + f(4) + f\left(\frac{16}{3}\right) + f\left(\frac{20}{3}\right) + f(8)\right) \approx 18.618$$

$$A_2 = \frac{4}{3}\left(f(0) + f\left(\frac{4}{3}\right) + f\left(\frac{8}{3}\right) + f(4) + f\left(\frac{16}{3}\right) + f\left(\frac{20}{3}\right)\right) \approx 15.951$$

$$A_{prom} = \frac{18.618 + 15.951}{2} = \boxed{\mathbf{17.284\ u^2}}$$

**3) *Resolver mediante sumas de Riemann***

$$\int_0^4 (x^2 - 3x)dx$$

*De la integral*:

$f(x) = x^2 - 3x$ , [0,4]

$$\Delta x = \frac{4-0}{n} = \frac{\mathbf{4}}{\boldsymbol{n}} \qquad x_k^* = 0 + \frac{4k}{n} = \frac{\boldsymbol{4k}}{\boldsymbol{n}}$$

$$f(x_k^*) = \left(\frac{4k}{n}\right)^2 - 3\left(\frac{4k}{n}\right) = \frac{\boldsymbol{16k^2}}{\boldsymbol{n^2}} - \frac{\boldsymbol{12k}}{\boldsymbol{n}}$$

$$A = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{16k^2}{n^2} - \frac{12k}{n}\right)\frac{4}{n} = \lim_{n\to\infty} \sum_{k=1}^{n} \left(\frac{64k^2}{n^3} - \frac{48k}{n^2}\right) =$$

$$\lim_{n\to\infty} \left(64 \sum_{k=1}^{n} \frac{k^2}{n^3} - 48 \sum_{k=1}^{n} \frac{k}{n^2}\right) = \lim_{n\to\infty} \left(64 \cdot \frac{n(n+1)(2n+1)}{6n^3} - 48 \cdot \frac{n(n+1)}{2n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{32(n+1)(2n+1)}{3n^2} - \frac{24(n+1)}{n}\right) =$$

$$\lim_{n\to\infty}\left(\frac{32(2n^2+n+2n+1)}{3n^2} - \frac{24n+24}{n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{64n^2+96n+32}{3n^2} - \frac{24n+24}{n}\right) =$$

$$\lim_{n\to\infty}\left(\frac{64n^2}{3n^2} + \frac{96n}{3n^2} + \frac{32}{3n^2} - \frac{24n}{n} - \frac{24}{n}\right) = \lim_{n\to\infty}\left(\frac{64}{3} + \frac{96n}{3n^2} + \frac{32}{3n^2} - 24 - \frac{24}{n}\right)$$

$$= \frac{64}{3} - 24 = \boxed{-\frac{\mathbf{8}}{\mathbf{3}}}$$

**4**) ***Hallar el valor de***:

$f(x) = 2x$ , *en el intervalo* $[0,2]$

*Expresamos en forma de integral definida*

$$= \int_0^2 2xdx = \frac{2x^2}{2}\Big|_0^2 = x^2\Big|_0^2 = (2)^2 - (0)^2 = \boxed{4}$$

**5**) ***Evaluar***:

$$\int_{-2}^{1} \frac{x}{(x^2+1)^2} dx$$

$u = x^2 + 1$

$du = 2xdx$

*si* $x = -2, entonces\ u = (-2)^2 + 1 = 4 + 1 = 5$

*si* $x = 1, entonces\ u = (1)^2 + 1 = 1 + 1 = 2$

*Entonces*

$$\frac{1}{2}\int_{-2}^{1}\frac{2x}{(x^2+1)^2}dx = \frac{1}{2}\int_{5}^{2}\frac{du}{u^2} = -\frac{1}{2}\int_{2}^{5}\frac{du}{u^2} = -\frac{1}{2}\int_{2}^{5}u^{-2}du = -\frac{1}{2}\cdot\frac{u^{-1}}{-1}\Big|_2^5$$

$$= \frac{1}{2u}\Big|_2^5 = \frac{1}{2(5)} - \frac{1}{2(2)} = \frac{1}{10} - \frac{1}{4} = \boxed{-\frac{\mathbf{3}}{\mathbf{20}}}$$